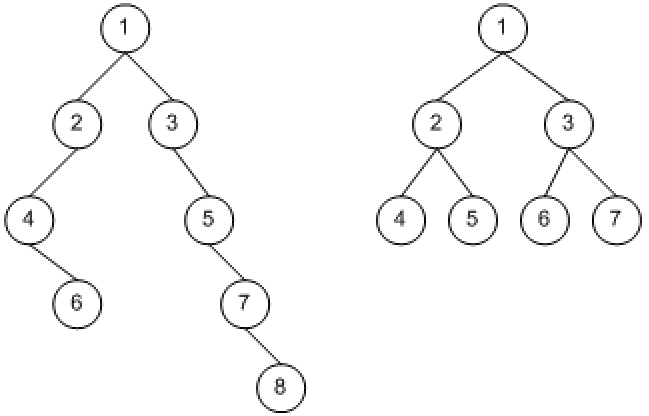

(a) (b)

# CSSE 230 Day 11

## Size vs height in a Binary Tree

After today, you should be able to…
… use the relationship between the size and height of a tree to find the maximum and minimum number of nodes a binary tree can have
…understand the idea of mathematical induction as a proof technique

http://i.msdn.microsoft.com/dynimg/IC71494.gif

# Team project starts next week

- Can voice preferences for partners for the term project (groups of 3)
  - EditorTrees partner preference survey on Moodle
    - Preferences balanced with experience level + work ethic
      - If course grades close, I'll honor mutual prefs.
      - If no mutual pref, best to list several potential members.
      - If you don't want to work with someone, I'll honor that. But if your homework or exam average is low, I will put you with others in a similar position. Sorry if that's not your preference, but I can't burden someone who is doing well with someone who isn't.
    - Consider asking potential partners these things:
      - Are you aiming to get an A, or is less OK?
      - Do you like to get it done early or to procrastinate?
      - Do you prefer to work daytime, evening, late night?
      - How many late days do you have left?
      - Do you normally get a lot of help on the homework?
  - Do it tonight, or by April 9.

# Other announcements

- HW4 late day extends until Saturday night (April 9). 8 days for the price of 1.
  - Good opportunity to catch up
  - Don’t blow it off
- Today:
  - Size vs height of trees: patterns and proofs
- Wrapping up the BST assignment, and worktime.

# Extreme Trees

- A tree with the maximum number of nodes for its height is a **full** tree.
  - Its height is O(log N)
- A tree with the minimum number of nodes for its height is essentially a ______ ______
  - Its height is O(N)
- Height matters!
  - Recall that the algorithms for search, insertion, and deletion in a binary search tree are O(h(T))

# To prove recursive properties (on trees), we use a technique called mathematical induction

- Actually, we use a variant called *strong induction* :

The former governor of California

# Strong Induction

- To prove that p(n) is true for all $n >= n_0$:
  - Prove that $p(n_0)$ is true (base case), and
  - For all $k > n_0$, prove that if we assume p(j) is true for $n_0 \leq j < k$, then p(k) is also true

- An analogy for those who took MA275:
  - Regular induction uses the previous domino to knock down the next
  - Strong induction uses all the previous dominos to knock down the next!

- Warmup: prove the arithmetic series formula
- Actual: prove the formula for N(T)